Assignment 1.

This homework is due *Tuesday*, September 13.

There are total 41 points in this assignment. 36 points is considered 100%. If you go over 36 points, you will get over 100% for this homework and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers sections 1.1–1.3 in Bartle–Sherbert.

(1) (Exercise 1.1.5 in textbook) For each $n \in \mathbb{N}$, let $A_n = \{(n+1)k \mid k \in \mathbb{N}\}$.
  (a) [2pt] What is $A_1 \cap A_2$?
  (b) [3pt] Determine sets $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$.

(2) Let $f : A \to B$ and $E, F \subseteq A$.
  (a) [3pt] (Part of 1.12) Show that $f(E \cap F) \subseteq f(E) \cap f(F)$.
  (b) [2pt] Show that not always $f(E \cap F) = f(E) \cap f(F)$. (*Hint:* as a counter-example, you can pick E and F that do not intersect *at all.*)
  (c) [2pt] Show that not always $f(E \setminus F) \subseteq f(E) \setminus f(F)$. (*Hint:* as a counter-example, you can pick $f(E)$ and $f(F)$ that *coincide.*)
  (d) [3pt] (Part of 1.13) Suppose, additionally, $G, H \subseteq B$. Prove that $f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$.

(3) [4pt] Let $f : A \to B$ and $g : B \to C$. Give "Cartesian product subset" (see def. 1.1.6 in textbook or def. of a function in lectures) definition of $g \circ f$: $g \circ f : A \to C$ is the following subset of $A \times C$: ...

(4) (1.1.20) Let $f : A \to B$ and $g : B \to C$.
  (a) [2pt] Show that if $g \circ f$ is injective, then f is injective. Give example that shows that g need not be injective.
  (b) [2pt] Show that if $g \circ f$ is surjective, then g is surjective. Give example that shows that f need not be surjective.

(5) [4pt] (Paragraph 1.2.4g) Find a mistake in the following (erroneous!) arguments:
Claim: If $n \in \mathbb{N}$ and if the maximum of the natural numbers p, q is n, then $p = q$.
"Proof." Proof by induction in n. Evidently, for $n = 1$ claim is true since in such case, $p = 1$ and $q = 1$.
Suppose, the claim holds for some $n \in \mathbb{N}$. Prove that then it also holds for $n+1$. Suppose maximum of p and q is $n+1$. Then maximum of $p-1$ and $q-1$ is $(n+1)-1 = n$. By induction hypothesis, $p-1 = q-1$, therefore $p = q$. Thus, the claim holds for $n+1$ and, by induction principle, for all natural numbers.

— *see next page* —

(6) [2pt] (1.3.5) Exhibit (define explicitly) a bijection from N to the set of all odd integers greater than 13.

(7) Exhibit (define explicitly) a bijection between
  (a) [2pt] $\mathbb{Z}$ and $\mathbb{Z} \setminus \{0\}$,
  (b) [3pt] $\mathbb{Q}$ and $\mathbb{Q} \setminus \{0\}$,
  (c) [3pt] $\mathbb{Q}$ and $\mathbb{Q} \setminus \mathbb{Z}$,

(8) [4pt] (1.3.12) Prove that the collection $\mathcal{F}(\mathbb{N})$ of all *finite* subsets of $\mathbb{N}$ is countable.